Circuit Analysis 1

Wire in a circuit allows any level of current to move without any voltage drop.

Current is a flow rate of charge through an element. Think of it like water, charges being its molecules.

Voltage is an electric potencial difference between two points. Imagin gravitational potencial differences of an object, the object here being the charge.

3 basic component of a circuit is resistor, capacitor and inductor. Often called as RLC.

Resistor is an element whose voltage is $\propto$ to current, also known as the ohm's law.

Capacitor is an element that stores electrical energy in an electrical field. It also opposes sudden change in voltage.

Inductor is an element that stores energy into a magnetic ~~f~~ field, this allows the element to oppose sudden changes in current.

$\cancel{Q = CV}$ $\quad \overset{V}{V} = \overset{A}{I}\overset{\Omega}{R} \quad I = \overset{F}{C}\frac{dv}{dt}$, $\cancel{C(F)}$ $\quad V = \overset{H}{L}\frac{di}{dt}$, $\cancel{L(H)}$

$$Q = CV$$

As with any other objects, the elements consumes or produce energy. The unit J, represents the energy delived to 1C charge by e-field with 1V voltage.

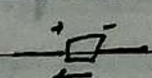
Power is ~~the~~ energy per time ~~of~~ And thus the power of an element in circuit can be derived.

$P = IV$ How ~~do~~ do we know if power is consumed or produced?

~~You your decided~~ Using potencial convention.

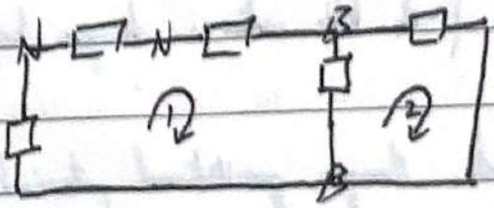

delivered ~~to~~ ~~by~~ to, ~~consumed~~

consuming / absorbing


delivered ~~to~~ by.

supplying

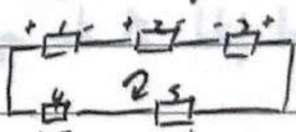
Geometry of circuit consists of :



Node, point of connection.
Branch, a place connecting two node.
Loop, node → branches → original node.

Two main laws governing circuit analysis are :

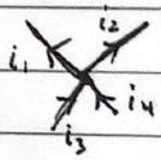
Kirchoff's voltage law, where sum of voltage in a given loop is zero.



$-V_1 - V_2 + V_3 - V_5 + V_4 = 0$

* note your polarity.
use potencial convention if possible.

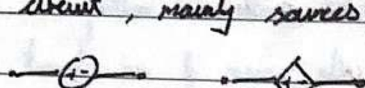
Kirchoff's current law, sum of current in & out is zero. in a given node / branch.



$-i_1 - i_2 + i_4 + i_3 = 0$

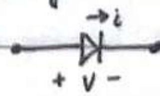
* in → pos out → neg.

More elements of the circuit, mainly sources:

Voltage Source  Ind Dep, Current Source  " "

Voltmeter tells the ~~voltage~~ voltage (b/w two terminal) at a place without effecting its current.
Ammeter current voltage.
Ohmmeter tells you the value of resistance of a given resistor.

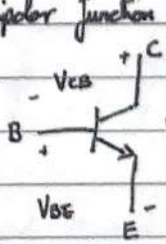
Simple overview of electronics:

Diode 

is a device that allows the current to flow only in one direction. When applied small +V, current is allowed to pass, however, when −V is applied, no current is allowed to pass. $V \geq 0 \quad i > 0$

$V < 0 \quad i = 0$

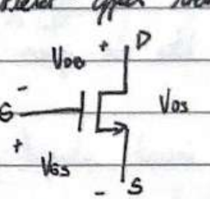
Bipolar Junction Collector is a device that in which current through collector (C) & emitter (E) is controlled by the base (B).



$i_C = \beta i_B \qquad i_E = i_C + i_B$

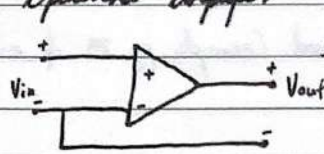
↳ current gain

Field-Effect Transistor is a device in which the current flowing in Source (S) & Drain (D) is controlled by the voltage applied in Gate (G)



$i_D \rightarrow$ depends on $V_{GS} \qquad i_S = i_G + i_D$

Operational Amplifier is a device that amplifies the difference in voltage between its two terminals.



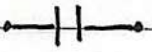
Introduction to 1st - order circuit :

1st order time dependent circuit is a circuit in which the behavior of voltage and current is described by 1st order linear ODE.

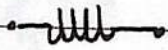
A systemic approach to 1st order circuit

1. KVL KCL element, before switching. Get desired variable.
2. Determine how the variable of interest behaves with switching action.
3. KVL KCL element, after switching. (a new circuit)
4. Eliminate variable to get 1st - order ODE
5. Use the info. on step 2 to solve the ODE.

Behaviors of C & L

 $i = C\frac{dv}{dt}$ During switching, capacitor will resist abrupt changes in voltage.

In DC-SS, capacitor will act like an open circuit.

 $v = L\frac{di}{dt}$ During switching, inductor will resist abrupt change in current.

In DC-SS, inductor will act like a short circuit, like on a wire.

Growth & decay of both components can be derived (example pg 25 of e-note).

Tau

τ is a time constant which the exponentially decaying function dropped to ≈ 0.36 to its original value.

$t > 0$ the general equation $V(t) = (V(0) - V(\infty))e^{-t/\tau} + V(\infty)$ is valid.

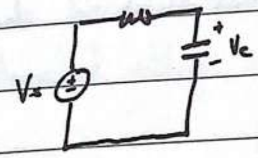
General solution for $V_C(t) = Ae^{-t/RC} + B$ where $\tau = RC$.

$V_L(t) = Ae^{-t/(\frac{L}{R})} + B$ where $\tau = \frac{L}{R}$

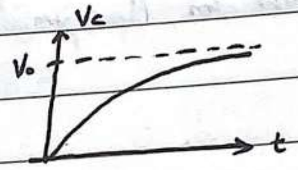
$\therefore V(t) = Ae^{-t/\tau} + B$ is the general solution for all first order circuit components.

Circuit Responses

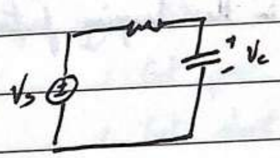
Step response is the response of a circuit to a step function.



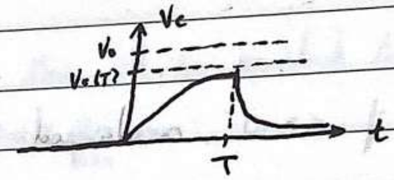
$V_s = V_0 u(t)$



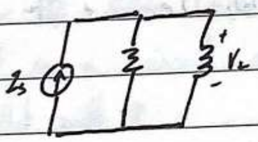
Pulse response is the response of a circuit to a pulse function.



$V_s = V_0 [u(t) - u(t-T)]$



Numerical Approach to TD circuit (finite difference time domain)



$I_s = I_0 u(t)$

$V_L = V_R = Ri_R = Ri_0$

$V_L' = \frac{-1}{L/R} V_L$

$V_L' + \frac{1}{L/R} V_L = 0$

using discretization

$V_L' \approx \frac{\Delta V_L}{\Delta t} = \frac{V_L(t+\Delta t) - V_L(t)}{\Delta t} = \frac{-1}{L/R} V_L(t)$

$V_L(t+\Delta t) = V_L(t) - \frac{\Delta t}{L/R} V_L(t)$

Introduction to 2nd Order circuit :

2nd order time dependant circuit, unlike the first, ~~consists of~~ come about when both component ~~R~~ C & L. The combination, then, provides a oscillating factor that exhibits a damping property.

Damping

For the equation $\frac{d^2x}{dt^2} + 2\alpha\frac{dx}{dt} + \omega^2 x = f$ where ω is the natural frequency

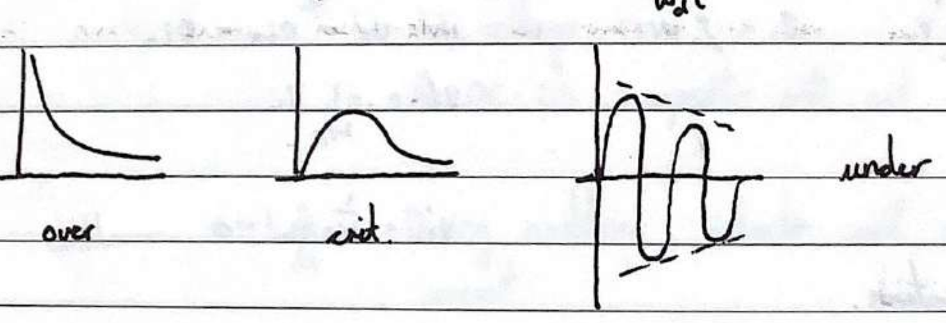
α is the damping factor

if $\alpha > \omega$, overdamped. $x(t) = M_1 e^{S_1 t} + M_2 e^{S_2 t} + \frac{f}{\omega^2}$

$S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega^2}$

if $\alpha = \omega$, critically damped $x(t) = (M_1 t + M_2)e^{-\alpha t} + \frac{f}{\omega^2}$

if $\alpha < \omega$, under damped $x(t) = (M_1 \cos(\omega_d t) + M_2 \sin(\omega_d t))e^{-\alpha t} + \frac{f}{\omega^2}$



Linear Circuit :

Linear circuit is a a circuit where all the components hold linear relationships. All a voltage and current will have linear dependance on all independant sources.

$V_j = \alpha_{1j} S_1 + \alpha_{2j} S_2 \ldots + \alpha_{Nj} S_j$

Superposition :

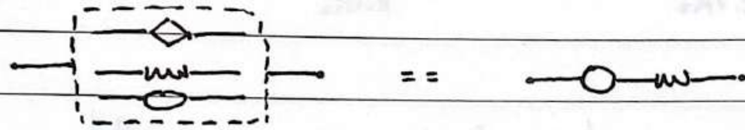
Superposition in a circuit describes that in a linear circuit you are able compute the sum of all independant source by computing indiviual parts and adding them.

Remember setting VS to zero → short, Using this we are able to compute indiviual

CS to zero → open sources V & I.

Thevenin Equivalent :

This theron says that if we have a circuit comprising of R, independ & dependant sources, in a linear relationship, we can represent any two point in a circuit by a independant voltage source and in series with R.

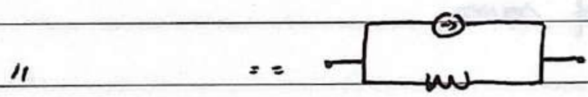


Steps you take

1. Get your open circuit voltage (V_{oc}). Thevin will be equivalent to V_{TH}.
2. Get short circuit current (I_{sc}). Note $R_{TH} = \frac{V_{TH}}{I_{sc}}$.

Norton Equivalent :

Under the same assumption & with Thevenin, this theron suggests that any two point in a circuit can be represented by independant current source in series parallel with R.



Steps also are identical.

To inter-change between the two represtation, use source transformation.

$V_{TH} = R_N I_N \rightarrow R_N = R_{TH} \quad \therefore \; V_{TH} = R_{TH} I_{sc}$

$\frac{V_{TH}}{R_{TH}} = I_N \qquad\qquad\qquad\qquad = R_N I_N$

Equivalent resistance is similar to Thevenin & Norton equivalent R_{TH}.

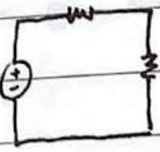
When ~~there~~ there's ~~not~~ no independant source, apply a voltage or current and compute the rest, divide the two ~~the~~ to get R.

$R_s = R_1 + R_2 \ldots + R_n$ (same current)

$R_{\parallel} = \left(\frac{1}{R_1} + \frac{1}{R_2} \ldots + \frac{1}{R_n}\right)^{-1}$ (same voltage)

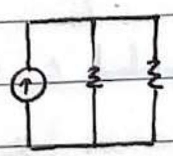
Circuit Divisions :

Voltage division allow a analysis of two R's in series.



$i = \frac{V}{R_1 + R_2}$ $V_1 = R_1 i = \frac{R_1}{R_1 + R_2} V$ $V_2 = R_2 i = \frac{R_2}{R_1 + R_2} V$

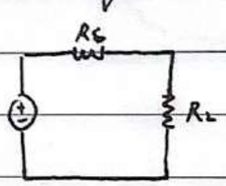
Current division allow a analysis of two R's in parallel.



$V = \frac{I}{\frac{1}{R_1} + \frac{1}{R_2}}$ $i_1 = \frac{V}{R_1} = \frac{R_2}{R_1 + R_2} I$ $i_2 = \frac{V}{R_2} = \frac{R_1}{R_1 + R_2} I_h$

Maximum Power :

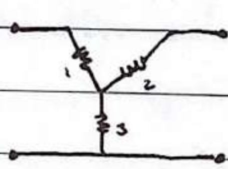
Maximum power theorem states that to draw a maximum power, resistance of load must equal resistance of source.

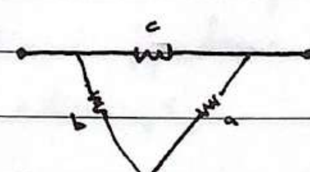


$P_{max} \rightarrow R_S = R_L$

$P_{max} = \frac{V_{TH}^2}{4R_{TH}}$

Wye - Delta Transformation :



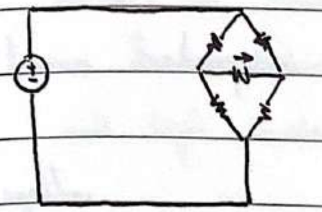


$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$ $R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$

$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$ $R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$

$R_3 = \frac{R_a R_b}{R_a + R_b + R_a}$ $R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$

The Wheatstone ~~Brogs~~ Bridge :

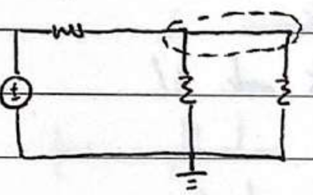


current through the middle is 0.

Nodal Analysis :

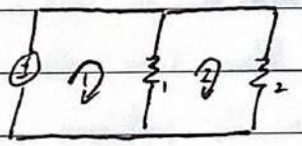
Node voltage is the voltage difference between a given node and a fixed reference node.

Defining any one of the nodes as a reference, with nodal voltages, we can compute any information in a given circuit.



Mesh Analysis :

Use the understanding that in a given loop, ~~current~~ voltage must add up. And the current within two loop through a element ~~...~~ co-exists in both must equal zero.



$i_1 = I_1 - I_2$ $\quad$ $M_1 : -V + V_1 = 0$ $\quad$ $V_1 = I_1 R_1$

$i_2 = I_2$ $\quad$ $M_2 : -V_1 + V_2 = 0$ $\quad$ $V_2 = I_2 R_2$

Super-position in larger circuits :

To find V_{oc}, solve V ~~by~~ ~~to~~ after killing independant voltage source and add the V you got killing the independant current source.

Also, utilize source transformations in getting the equivalent circuits.

~~Mesh Analysis Inspection~~:

Analysis Inspection :

Nodal inspection can be made when all sources in a circuit is independent current source.

Mesh inspection " voltage source.

$$\begin{bmatrix} G_{11} & \cdots & G_{1N} \\ \vdots & & \vdots \\ G_{N1} & \cdots & G_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix}$$

Nodal - G_{KK}, sum of conductance ($\frac{1}{R}$) at node K.

G_{KL}, neg. of sum of the conductance at node KL.

V_K, unknown voltage at node K.

I_K, sum of indep. current source at node K.

Mesh - R_{KK}, " resistance " mesh K

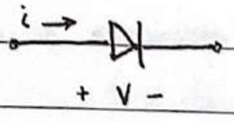
~~G~~ R_{KL}, " resistance " mesh KL

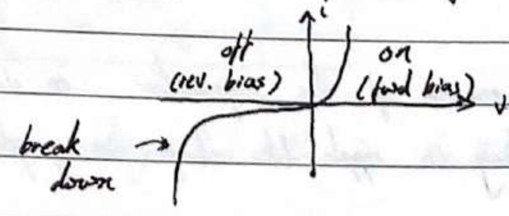
I_K, " current " mesh K

V_K, " voltage " mesh K.

Introduction to Electronics :

Diodes are element in which at a positive input voltage, it behaves with a low resistance allowing the current to go through and at ~~high resistance~~ neg. voltage, it behaves with high ~~resstory~~ resistance, allowing no current to go through.





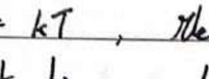
$i = I_s \left(e^{\frac{V}{nV_{TH}}} - 1 \right)$

n, ideality factor

~~V_{TH}, kT~~ $V_{TH} = kT$, the Boltzmann constant · Absolute temp.

I_s, reverse saturation current.

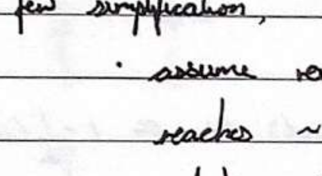
few simplification,

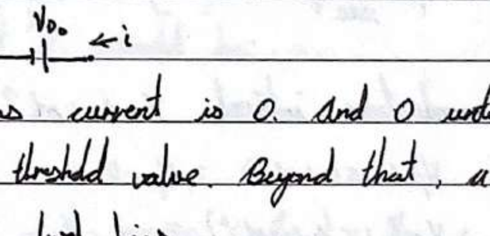


- assume reverse bias current is 0. And 0 untill pos. voltage reaches ~0.7V of threshold value. Beyond that, a linear increase.
- neglect resistance at fwd bias.
- turn-on voltage is 0.

How do we know in a given circuit that a diode is fwd or rev?
We can't untill we try every possibilities and find the right one with computing V's on an assumptions.

Rectification,





* V_{out} has pos. avg.

$V_{in} = V_D + V_{out}$

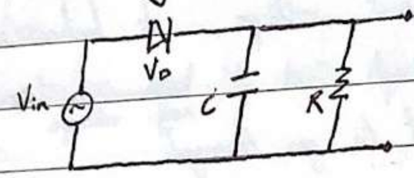
$V_{in} > 0$, $V_D = 0$

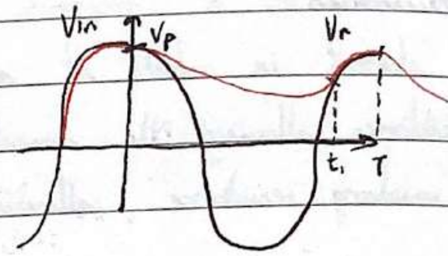
$V_{in} < 0$, $V_{out} = 0$

$T = \frac{1}{f}$

$\omega = 2\pi f$

Filtering,





Due to the presense of the capacitor, a voltage experiences an exponencial decay, creating a ripple-like shape in graph.

When $V_r \ll V_p$,

$$(T \gg RC) \quad V_r \approx V_p(1-e^{-\frac{T}{RC}})$$

$T \ll RC$,

$$V_r \approx \frac{T}{RC} V_p$$

$$V_{avg} \approx V_p - \frac{T}{2RC} V_p \qquad i_{avg} = \frac{V_{out,avg}}{R}$$

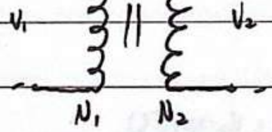
The conduction interval $T - t_1 = \Delta t$?

$$V_p(\cos(\omega \Delta t)) = V_p - V_r \qquad \cos(\omega \Delta t) \ll 1 \approx 1 - \frac{1}{2}(\omega \Delta t)^2$$

$$V_p(1 - \tfrac{1}{2}(\omega \Delta t)^2) \approx V_p - V_r$$

$$\Delta t \approx \frac{T}{2\pi}\left(\frac{2TV_p}{RC}\frac{1}{V_p}\right)^{\frac{1}{2}}$$

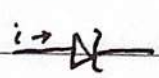
Transformer,



Device that transfers electric energy from one circuit to another, either increasing or decreasing the circuit voltage.

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

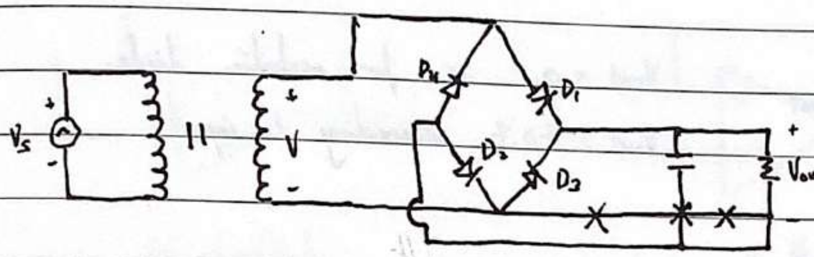
Zener Diode,



Allow forward flow in a similar manner with ideal diode but also permit reverse flow once above a value so known as breakdown voltage, where the diode provide a constant output voltage regardless on the changes in the are input.

$V_Z = V_{Z0} + r_Z I_Z$ At reverse $V_{Z0} = 8V$ $r_Z = 20\Omega$ $I_{ZK} = 1mA$

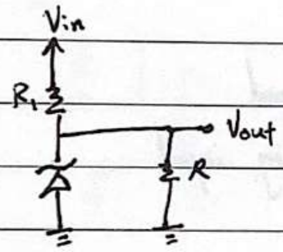
~~Ref~~ Rectification, Filtering & Transforming:



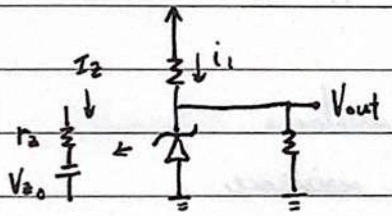
$V > 0$, D_1 & D_2 ~~is~~ are on making $V_{out} = V$ as V_{D1} & $V_{D2} = 0$.

$V < 0$, D_3 & D_4 are on making $V_{out} = -V$ as V_{D3} & $V_{D4} = 0$

Regulation:



Want to deliver a certain voltage to R with Vin being a sinusoidal input. This means we need to know what R_1 should be.



We must ensure $I_{ZK} < I_Z <$ ~~A~~ I_{Zmax}.

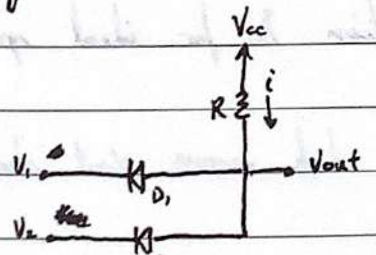
for $I_{ZK} < I_Z$,

Compute i_1 when V_{in} is at min and get R_{1max}.

for $I_Z < I_{Zmax}$,

Compute i_1 when V_{in} is at max and get R_{1min}.

Logic Gates:



If $V_1 = V_{cc}$ $V_2 = V_{cc}$,

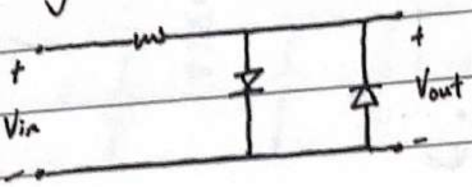
D_1 & D_2 off $\rightarrow i = 0 \rightarrow V_{out} = V_{cc}$

If $V_1 = 0$ $V_2 = V_{cc}$

D_1 on $\rightarrow V_1 = V_{out} \rightarrow V_{out} = 0$

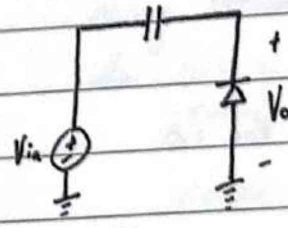
$\therefore$ AND gate.

Limiting :



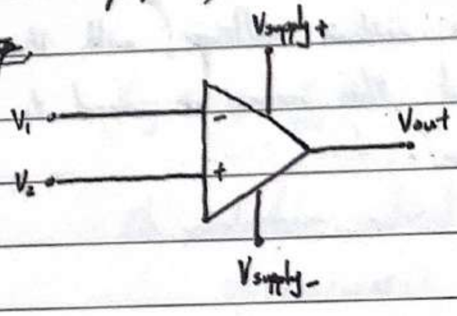
$V_{out} = 0$ or for realistic diode,
$V_{out} = \pm 0.7$ according to input.

Clamping :

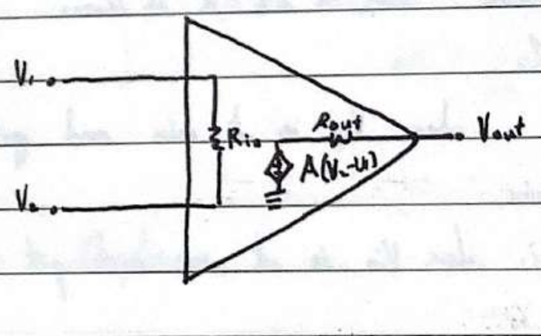


Adding DC value to AC input
(shifting waves in y-axis)

Operation Amplifier,
Opamp,



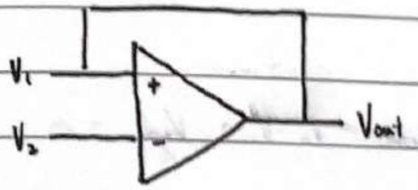
V_1 - Inverting input
V_2 - Non-Inverting input



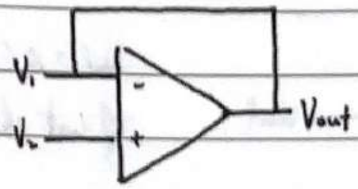
Ri - Input resistance
Rout - Output resistance
A - Open loop gain

Operation Amplifier amplifies voltages.
We want high Rin & low Rout for optimal amplification. So for ideal opamp,
$R_i = \infty$, $R_{out} = 0$, $A = \infty$
We also want ~~at Vout~~ Vout stay within linear region, which means Vout should be bounded to Vsupply's.

positive & negative feedback :



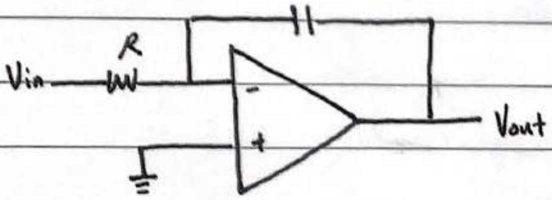
pos feedback



neg feedback ($V_+ = V_-$)

Other applications of opamps :

Integrator



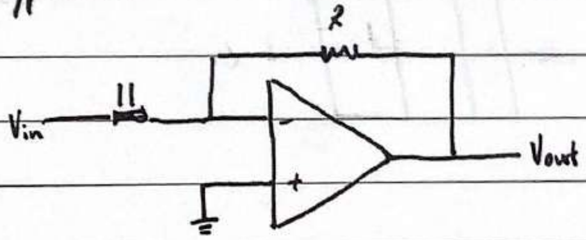
$i_c = C\frac{dV_c}{dt}$ $\qquad i = \frac{V_{in}}{R}$

$V_c = \int_0^{V_c} dV_c = \frac{1}{C}\int_0^t i\,dt$

↳ initially uncharged.

$$V_{out} = \frac{-1}{RC}\int_0^t V_{in}\,dt$$

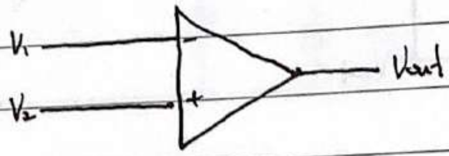
Differenciator :



$i_c = C\frac{dV_c}{dt} = C\frac{dV_{in}}{dt}$

$V_{out} = -RC\frac{dV_{in}}{dt}$

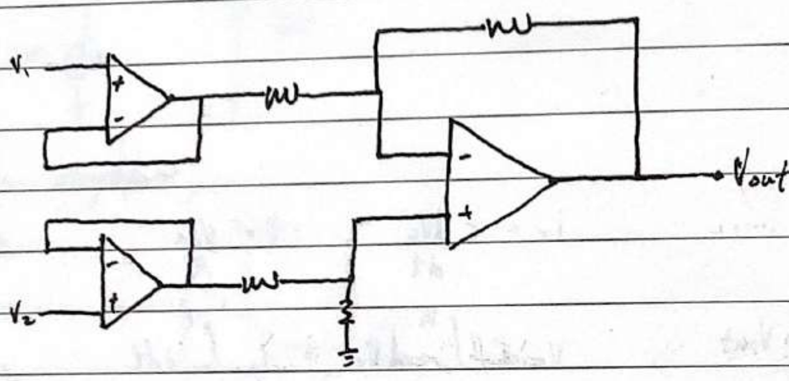
Ideal Op-Amps:



$V_{out} = A(V_2 - V_1)$

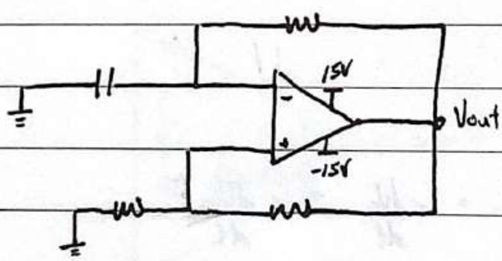
If at linear region $V_2 - V_1 = \frac{V_{out}}{A} = 0$

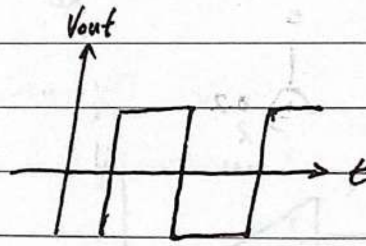
Instrumental Amp:



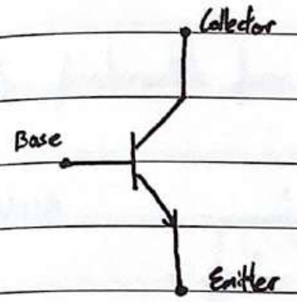
Cascading op-amp circuits.

Oscillator:





Bipolar Junction Transistor



NPN

$i_E = i_C + i_B$

$i_C = \beta i_B$

$= \alpha i_E$

$\alpha = \dfrac{\beta}{\beta + 1}$

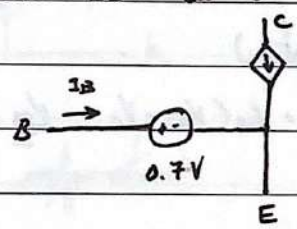
$i_C \approx I_S e^{\left(\frac{V_{BE}}{V_T}\right)}$

↳ saturation current

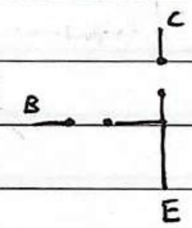
3 models to represent BJT depending on I_B & V_{CE}.

Steps:

represent BJT as a linear model

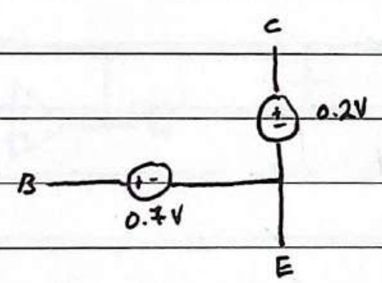


if $I_B < 0$, cutoff model



if $I_B > 0$ _,

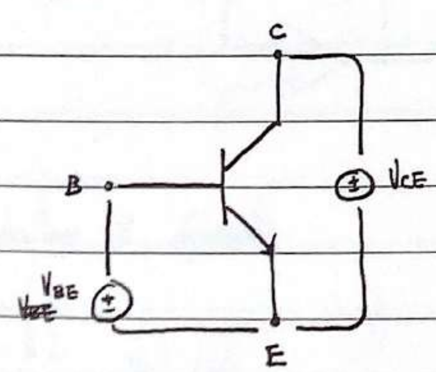
test for saturation



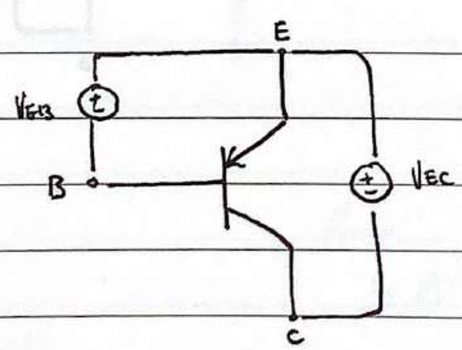
if $V_{CE} \leq 0.2$, saturation model

if $V_{CE} > 0.2$, linear model

PNP & NPN

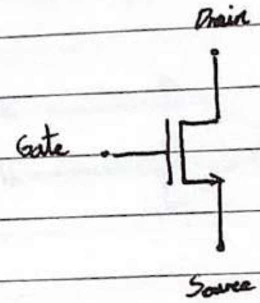


NPN



PNP

Metal Oxide Semiconductor Field Effect Transistor



width — oxide cap per unit area.

$$k_n = \frac{W}{L}\mu C_{ox}$$

channel length — mobility

n-channel

over-drive voltage

3 Ways to represent MOSFET V_{GS} & V_{ov}

If $V_{GS} < V_t$, cut off so $I_D = 0A$

If $V_{GS} > V_t$, compute $V_{ov} = V_{GS} - V_t$

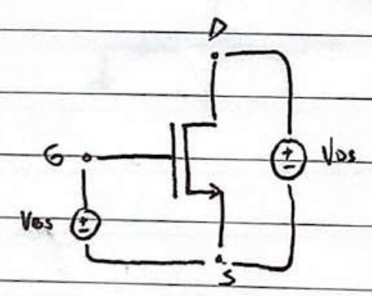
if $V_{DS} < V_{ov}$, triode (variable resistor)

$$g_{DS} = k_n\left(V_{GS} - V_t - \frac{V_{DS}}{2}\right), \quad I_D = k_n\left(V_{GS} - V_t - \frac{V_{DS}}{2}\right)V_{DS}$$

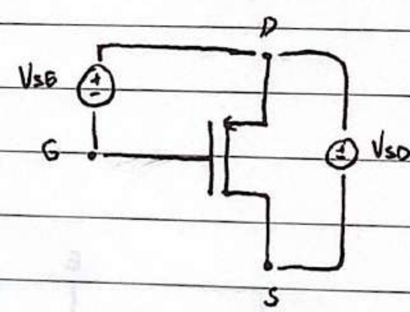
if $V_{DS} >= V_{ov}$, saturation (linear)

$$I_D = \frac{1}{2}k_n(V_{GS} - V_t)^2$$

n & p channels



n-channel

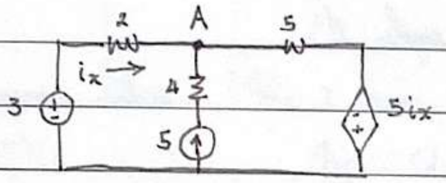


$V_{SG} - |V_t|$ ~~V_{ov}~~ shift to this.

Circuit Analysis II

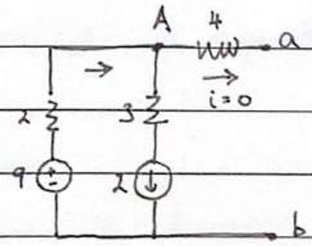
Few fundamentals from the previous:

MNA



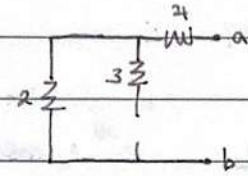
$$\frac{3-A}{3} + 5 = \frac{A+5i_x}{5} \quad , \quad i_x = \frac{3-A}{5}$$

Thevenin



$$\frac{9-A}{2} = 2$$

$A \rightarrow V_{TH}$



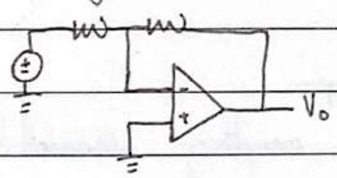
short VS

open CS

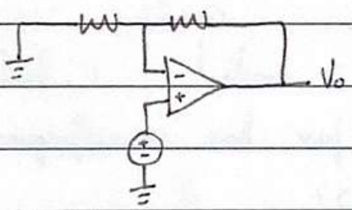
$R_{TH} = 6\ \Omega$

Op-Amps (basic configuration)

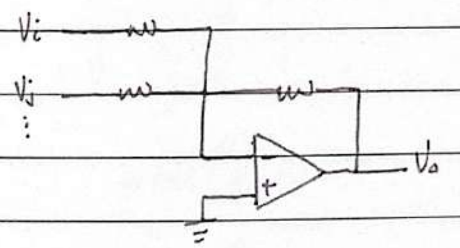
Inverting



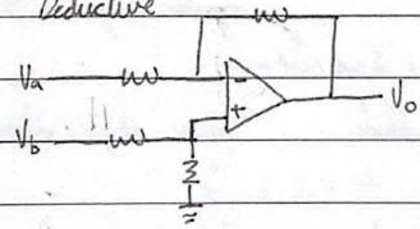
Non-inverting



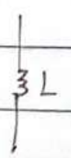
Additive



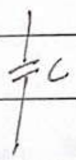
Deductive



Inductors & Capacitors



$$V_L = L\frac{di}{dt}$$

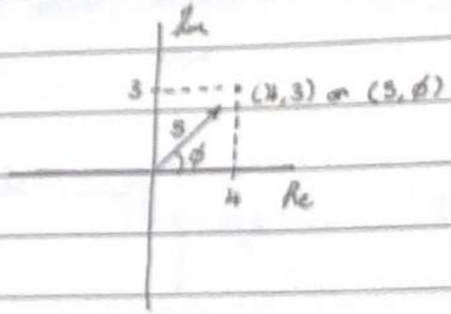


$$I_C = C\frac{dv}{dt}$$

Power

$$P = IV$$

Complex Numbers :



$4+3j$ or $5\angle\phi$

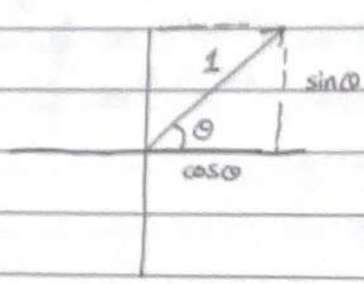
arithmetics of complex #'s

add & sub → real & im , vector representation

mult. → $(a+jb)(c+jd)$

div. → $\frac{a+jb}{c+jd} = \frac{(a+jb)(c-jd)}{(c+jd)(c-jd)}$

euler's #



$e^{j\theta} = \cos\theta + j\sin\theta$ ← radians

$ae^{j\theta}$ ← angle, ↑ length $a\angle\theta = \cos\theta + i\sin\theta$

Steady State :

DC, long-time (SS) → everything is constant → L short C open

AC, " → sinusoidal func. have same frequency → everything sinusoidal →

LRC is represented by ohms (Ω)

The sine function, $y = A\sin(\omega t + \theta)$ — A: amplitude, ω: angular freq. (rad/s), θ: phase shift (radians)

steinmetz theorem (of add & sub sinusoidal functions)

$\underbrace{A\cos(\omega t + \alpha) + B\sin(\omega t + \beta)} = C\cos(\omega t + \gamma)$

adding $Re(Ae^{j(\omega t+\alpha)}) + Re(Be^{j(\omega t+\beta)}) = Re(Ce^{j(\omega t+\gamma)})$

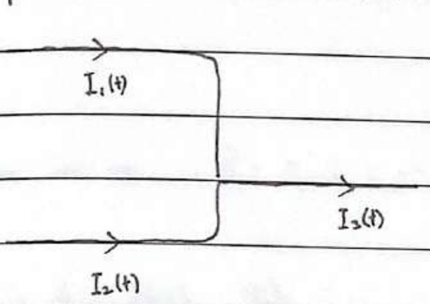
$Ae^{j(\omega t+\alpha)} + Be^{j(\omega t+\beta)} = Ce^{j(\omega t+\gamma)}$

$e^{j\omega t}(Ae^{j\alpha} + Be^{j\beta}) = e^{j\omega t}(Ce^{j\gamma})$ ← complex # representation of sinusoid

or $A\angle\alpha + B\angle\beta = C\angle\gamma$ Phasors.

KVL & KCL

Just like in DC circuit, we can use the circuit analysis skill like MNA.



$i_1(t) = 5\cos(300t + 30°)$ → $i_3(t) = 5∡30 + 7∡-20°$

$i_2(t) = 7\cos(300t - 20°)$ $= 10.908∡0.556°$

or

$10.907 + 0.105i$

$= 10.91\cos(300t + 0.556°)$

$\downarrow$

$\frac{0.556\pi}{180}$

to compute instantaneous i_3.

Impedance:

Resistance (Ω) of components in AC circuit.

Derivation

say $i(t) = A\cos(\omega t + \alpha) \longleftrightarrow A∡\alpha$

$i'(t) = -\omega A\sin(\omega t + \alpha)$

$= \omega A\sin(\omega t + \alpha + 90°) \rightarrow \omega A∡90°$

or

$j\omega(A∡\alpha)$

$\int i(t) = \frac{A}{\omega}\cos(\omega t + \alpha - 90°) \rightarrow \frac{A}{\omega}∡-90°$

or

$\frac{A∡\alpha}{j\omega}$

Inductor

$V_L = L\frac{di}{dt}$

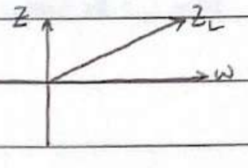
$= Lj\omega(A∡\alpha)$

$\underbrace{\bar{V}_L}_{\bar{V}} = \underbrace{j\omega L}_{\Omega_L}\underbrace{\bar{I}_L}_{\bar{A}}$

$\downarrow$

Z_L & $X_L = \omega L$

impedance reactance



Capacitor

$i_c = C\frac{dV}{dt}$

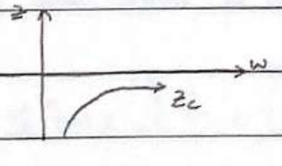
$= Cj\omega(A∡\alpha)$

$= j\omega C\bar{V}_c$

$\bar{V}_c = \frac{1}{j\omega C}i_c = -j\frac{1}{\omega C}i_c$

$\downarrow$

Z_c & $X_c = -\frac{1}{\omega C}$



Resistor

$\bar{V}_R = R\bar{I}_R$

Ohm's Law

$\bar{V} = \bar{Z}\bar{I}$, where $Z_L = j\omega L$ $Z_c = -j\frac{1}{\omega C}$ $Z_R = R$

Phase Implication of Ohm's Law

Inductor 'stretches' current phasor by ωL times.

Capacitor 'shrinks' " ωC times.

Power in AC :

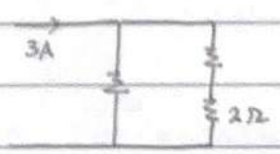
Power for AC circuit requires different implecation to DC circuit.

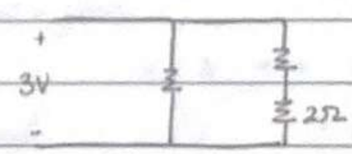
DC Review

$P = VI$

Power in Resistor (only resistor!)

$P_R = I^2 R = \frac{V^2}{R}$





$P_R \neq 2(3)^2$ as $I_{2\Omega} \neq 3A$ $\qquad$ $P_R \neq \frac{3^2}{2}$ as $V_{2\Omega} \neq 3V$

Root Mean Square

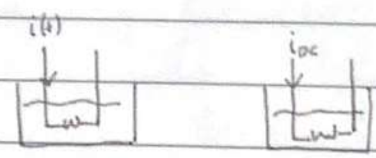
DC (A) reads I_{av} → for $I(t)_{avg} = 0$ as sin & cos.

To make the reading work → take square & $\sqrt{\ }$

$\therefore$ at AC, $I_{DC} = \left(\frac{1}{T}\int_T f(t)^2 dt\right)^{1/2}$, AC (A) → I_{RMS} AC (V) → V_{RMS}

Effective Value

Analogy,



where heating = P_{avg} dissipated.

when $i(t)$ & i_{DC} cause the water to be same temp at the same time, the value of I is denoted as I_{eff} (or I_{DC}).

$I_{DC} \rightarrow P = R I_{DC}^2$ $\qquad$ $I_{AC} \rightarrow P = \frac{1}{T}\int_0^T R\, i(t)^2 dt$

Effective Value for Sinusoids

$i(t) = I_{peak} \sin(\omega t + \theta) \Rightarrow I_{eff}$ or $I_{DC} = \sqrt{\frac{1}{T}\int_0^T I_{peak}^2 \sin^2(\omega t + \theta)\,dt}$

$= \frac{I_{peak}}{\sqrt{2}}$

From now on ... $\bar{I} = I_{peak} \angle 0° \Rightarrow \bar{I} = I_{rms} \angle 0°$
rms angle

Power in AC Steady State :

Generalization of I_{eff}

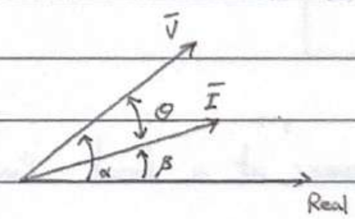
$i(t) = A\sin(\omega t + \theta)$ heat a resistor the same as I_{DC} with a value of $\frac{A}{\sqrt{2}}$.

If we call 'M' effective value of $i(t)$, $M = \frac{A}{\sqrt{2}}$. Then, $i(t) = \sqrt{2}\,M \sin(\omega t + \theta)$

Active & Reactive & Apparent

Voltage in generic load is $v(t) = \sqrt{2}\,V \sin(\omega t + \alpha)$ if $i(t) = \sqrt{2}\,I \sin(\omega t + \beta)$

$\bar{V} = V_{rms} \angle \alpha°$ $\qquad \bar{I} = I_{rms} \angle \beta°$



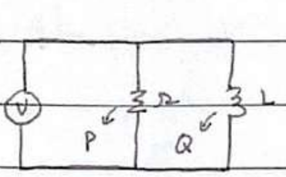
$p(t) = i(t)v(t) = 2VI \sin(\omega t + \alpha)\sin(\omega t + \beta)$ simplifying ...

$= VI\cos(\alpha - \beta) + VI\cos(2\omega t + \alpha + \beta)$

$P_{avg} = VI\cos(\alpha - \beta) - 0$

$= V_{RMS} I_{RMS} \overbrace{\cos(\underbrace{\alpha - \beta}_{\theta,\ \text{power factor angle}})}^{\text{power factor (\%), inductive (lagging) \& capacitive (leading)}}$ } Active Power (W), positive source → load

$Q = VI\sin(\alpha - \beta)$ } Reactive Power (VAr), load → source (phenomena in all non-linear) power that goes back & forth without doing meaningful work.



Apparent Power $S = VI$ (VA)

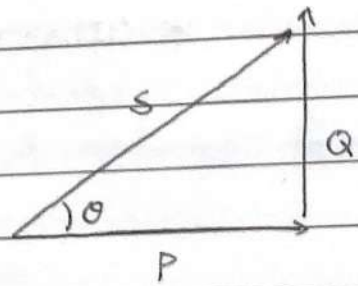
$R \Rightarrow \theta = 0 \quad P = S \quad Q = 0$

$L \Rightarrow \theta = 90° \quad P = 0 \quad Q = S$

$C \Rightarrow \theta = -90° \quad P = 0 \quad Q = -S$

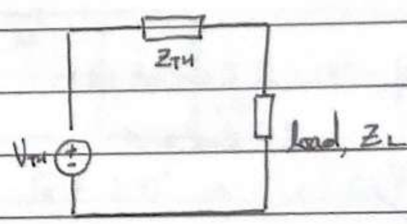
$S = P + Qj$; $Q = (S^2 + P^2)^{\frac{1}{2}}$ for power triangle

$P = S\cos\theta = S \cdot pf$ $\quad Q = S\sin\theta$; Powers can also be computed using $Z|I|^2$ & $\frac{|V|^2}{Z}$



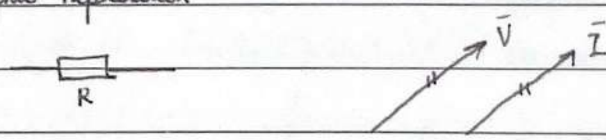
Maximum Powers

To obtain Pave max, V_{TH} & Z_{TH} must be established where $Z_L = Z_{TH}^*$

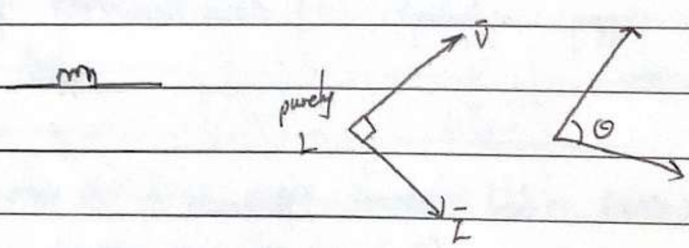


The maximum complex power at a load $P_{max} = \frac{|V_{TH}|^2}{8Z_{TH}}$

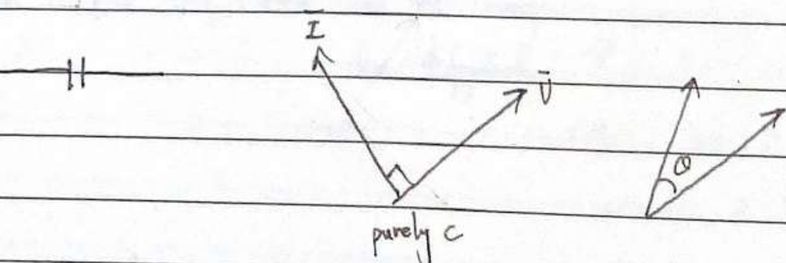
Graphic Representation



no diff $\quad 2\Omega$

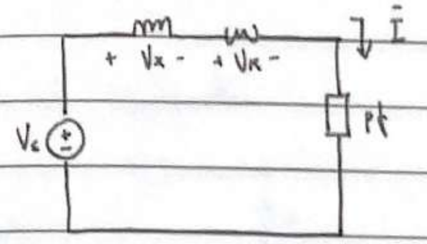


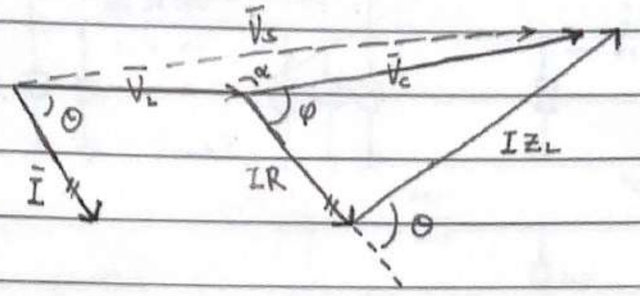
I lag V $\quad 2 + 3j\ \Omega$ pos.



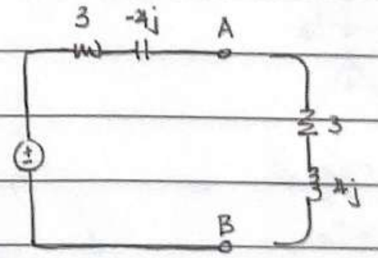
I lead V $\quad 2 - 3j$ neg.

One Common Circuit

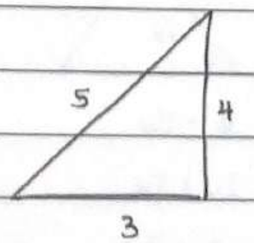




$\alpha = 180 - (\varphi - \theta)$ where $\varphi = \tan^{-1}\left(\frac{Z_L}{R}\right)$



→



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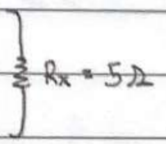
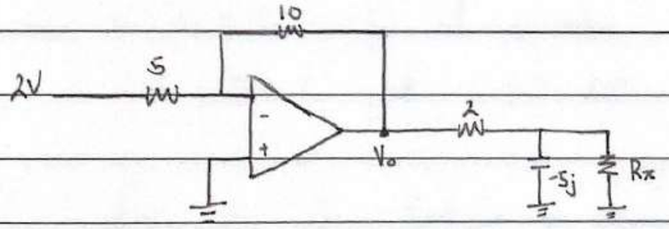


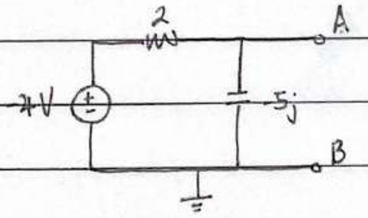
$R_x = |\bar{Z}_{TH}|$

Op-Amp.

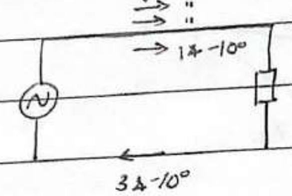


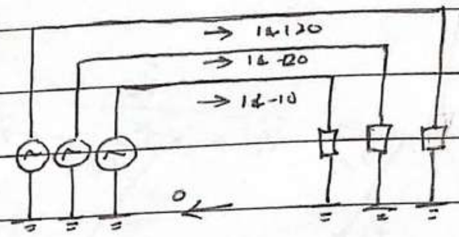
$V_o = -\frac{10}{5}(2) = -4$

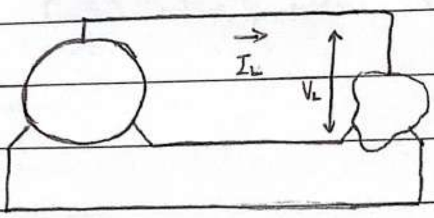
↓

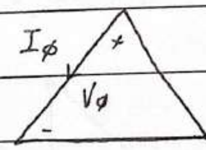


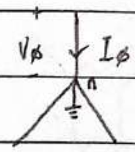
Three Phase System :







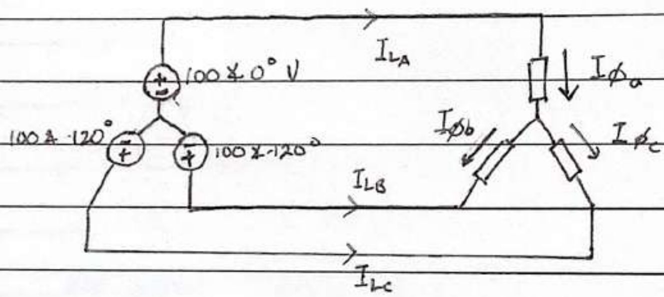




$V_L = V_\phi$ $\qquad$ $V_L \neq V_\phi$

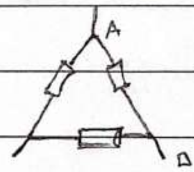
$I_L \neq I_\phi$ $\qquad$ $I_L = I_\phi$

$I_L = \sqrt{3}\, I_\phi$ $\qquad$ $V_L = \sqrt{3}\, V_\phi$



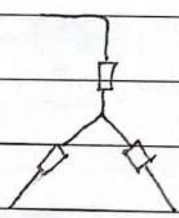
$3R_Y \rightarrow$

$\frac{1}{3} R_\Delta \leftarrow$



also $\quad V_{AB} = V_{AN} + 30°$

Power in 3 phase systems

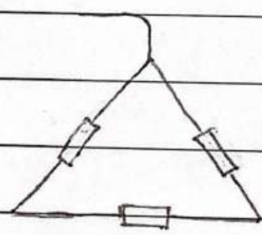


$P = V_\phi I_\phi \cos\theta_\phi$

$P_{3\phi} = 3 V_\phi I_\phi \cos\theta_\phi$

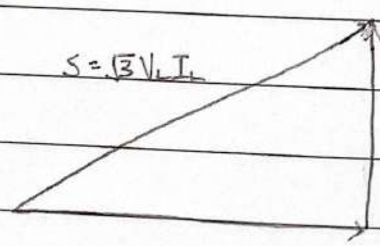
$V_\phi = \dfrac{V_L}{\sqrt{3}}$ $\qquad$ $I_\phi = I_L$

$= \sqrt{3}\, V_L I_L \cos\theta_\phi$



$P_{3\phi} = \sqrt{3}\, V_L I_L \cos\theta_\phi$

$S = \sqrt{3}\, V_L I_L$

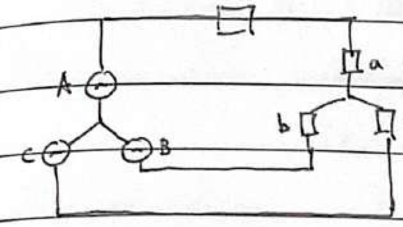


$Q = \sqrt{3}\, V_L I_L \sin\theta_\phi$

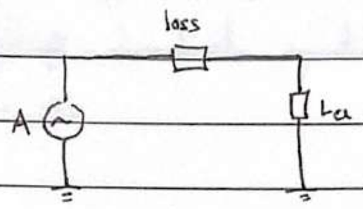
$P = \sqrt{3}\, V_L I_L \cos\theta_\phi$

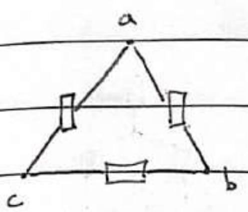
Power at the load $\Rightarrow \bar{S} = |\bar{I}|^2 \bar{Z}$

Conversion and Simplification



can be represented



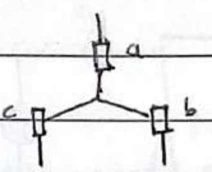


$\rightarrow$ $1/3$ for load

$I_L = I_{aY}$ $\quad I_L = \sqrt{3}\, I_{a\Delta}$

$I_{aY} = \sqrt{3}\, I_{ab} \measuredangle\, I_{ab}^{\circ} - 30^{\circ}$

$\rightarrow$



$V_L = \sqrt{3}\, V_{aY}$ $\quad V_L = V_{ab}$

$V_{aY} = \frac{V_{ab}}{\sqrt{3}} \measuredangle\, V_{ab}^{\circ} - 30^{\circ}$

Power, $S = \sqrt{3}\, V_L I_L$ is for real rms value.

$\therefore \bar{S} = \sqrt{3}\, V_L I_L$ with $\measuredangle\, V_L^{\circ} - I_L^{\circ}$ or $S = 3 V_{ph} I_{ph}^*$ $\nearrow$ but often we don't have this info IRL.

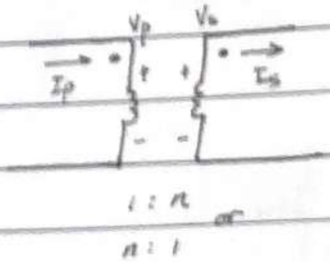
also, S_L where load is in series (Y), S_L seen by the source $= 3S_L$

S_L " parallel (Δ), " $= S_L$

$S_L = |I_L|^2 Z_L$ or $V_L I_L$, $\bar{S}_L = V_L I_L^*$ ...

1 Transformers :

A device that adjust the V I P according to the device's coefficient (transformation ratio)



Step - Down [n : 1 , n > 1]

$V_p = nV_s$

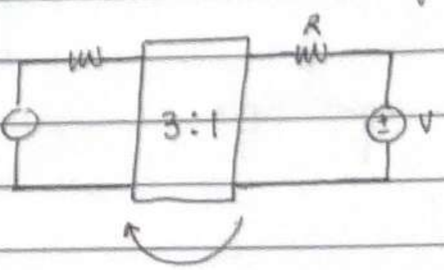
$nI_p = I_s$

Step - Up [1 : n , n > 1]

$nV_p = V_s$

$I_p = nI_s$

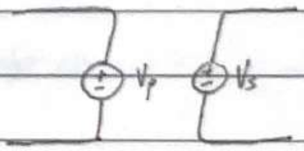
We can move circuit elements from one side to the other.



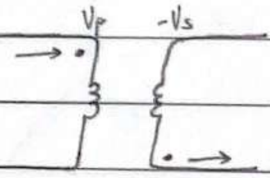
a^2R , aV , I/a Low - to - high n : 1

R/a^2 , V/a , aI High - to - low 1 : n

Treating each transformer winding as a voltage source, simplifies many analytical steps.



Dot - Convention :



current goes in at one side, must come out at other.

Notice that power equation stays the same, $S = V_{rms} I^*_{rms}$

* $I_p = \sum I_{sk}$ Primary current is the combination of sum of all secondary currents.

2^{nd} - Order Circuit :

The order in a circuit represents # reactive elements in a circuit.
∴ circuit with 2 reactive element.

Solving the circuit, we make use of p-operator ($p = \frac{d}{dt}$) & characteristic equations for ODE.

Steps for solving :

1. Find conditions $t < 0$, using usual analysis tools.
2. Find conditions $t > 0^+$, using $i_L'(0^+) = \frac{V_L(0^+)}{L}$ $V_c'(0^+) = \frac{i_c(0^+)}{C}$ with reactive values from step 1 as $\underset{V\text{-}Src}{V_c(0^+) = V_c(0^-)}$ & $\underset{I\text{-}Src}{i_L(0^+) = i_L(0^-)}$.
3. Find the equation for asked variable by converting the $\underline{t > 0}$ circuit to p-operatable using $R \Rightarrow R$ $L \Rightarrow Lp$ $C \Rightarrow \frac{1}{Cp}$.
4. Make the characteristic equation from the denominator of the eq. at step 3, and get eigen-values. This allows us to know with solution to ODE is most fit, and the time-to-steady state $= 5\tau_{larger}$
   $\hookrightarrow$ $\tau_1 = -\frac{1}{b_1}$ $\tau_2 = -\frac{1}{b_2}$
5. Use ODE solution, b_1 & b_2

$b^2 > 4ac$ b_1 & $b_2 \in \mathbb{R}$ (Over-damped) — $y(t) = k_1 e^{b_1 t} + k_2 e^{b_2 t} + k_3$

$b^2 < 4ac$ b_1 & $b_2 \notin \mathbb{R}$ (Under-damped) — $y(t) = k_1 e^{\sigma t} \cos(\omega t + k_2) + k_3$, $s = \sigma + \omega j$

$b^2 = 4ac$ $b_1 = b_2$ (Critically-damped) — $y(t) = (k_1 t + k_2)e^{bt} + k_3$

And the IC's, $y(0)$ & $y'(0)$, to solve the coefficients k_1 & k_2 & k_3.

Note,

- $(0.1p^2 + 17p + 100)V_c = \overset{0}{\cancel{2p}} + 600$ & $k_3 = \frac{600}{100} = 6$.
- For U-D case, one can also use

$y(t) = e^{\sigma t}[k_1 \cos(\omega t) + k_2 \sin(\omega t)]$

$y'(t) = e^{\sigma t}[(\omega k_2 - \sigma k_1)\cos(\omega t) - (\omega k_1 + \sigma k_2)\sin(\omega t)]$

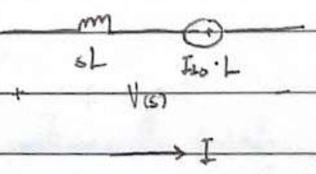
To laplace:

$A\cos(\omega t + \theta) \rightarrow \underset{\text{circuit}}{\boxed{}} \rightarrow kA\cos(\omega t + (\theta + \phi))$ $\quad kA\angle(\theta+\phi) = \underset{\sim}{k\angle\phi} \cdot A\angle\theta$

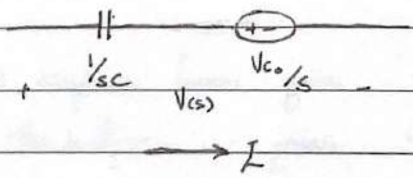
↳ effect of the circuit to signal.

With initial conditions, we can transform the circuit in laplace domain $\underline{t>0}$.

Inductor



Capacitance



Solve the circuit using 'normal' & 'ilaplace' function in HP Prime by solving for the right equation. An equation for a given parameter can be solved by taking the inverse-laplace.

Note that maximum of something can be solved using graph, or taking the 0's of derivative.

Transfer - Function :

$Y(s) = H(s)\,X(s)$ → input function in s-domain

↓ output ↳ tf, behavior of circuit $H(s) = \dfrac{Y(s)}{X(s)}$ at energy 0 [V_{C0} & $I_{L0} = 0$, Vsrc short, Isrc Open]

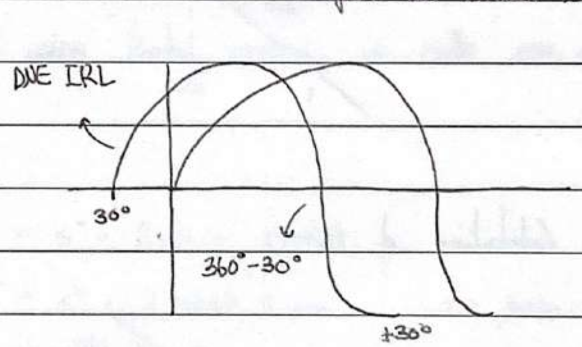
$|H(s)| = k$ $H(s)^\circ = \delta$, $k\,(dB) = 20 \log_{10}(k)$ $\delta\,(rad) = \dfrac{\delta \cdot 180}{\pi}$

Notice if $X(s)$, the input is dirac-delta $\delta(x)$, $H(s) = Y(s)$

Time delay of the signal can be computed using δ, $\dfrac{(\delta(rad) - 2\pi)\cdot \frac{2\pi}{\omega}}{2\pi}$ or $\dfrac{(\delta(deg) - 360)\cdot \frac{2\pi}{\omega}}{360}$

* if pole @ RHS ⇒ unstable.

* but δ here must come before the output, ∴ -2π (if rad) or -360 (if °) from the circuit signal $H(s)$.



Bode Plot:

We simulate the frequency response of a circuit using Bode plot (both amplitude & degrees).
(dB) (deg)

Amplitude Plot

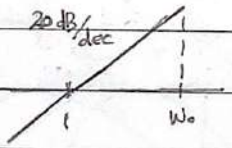
Decibel of any given frequency can be computed using $20\log_{10}(\omega)$ where $\omega\,(\text{rad}/s) = 2\pi f$.

Given a $H(s) = k \cdot \dfrac{s(s+a)(s+b)^2}{(s+c)(s+d)}$

Contribution of a constant k,

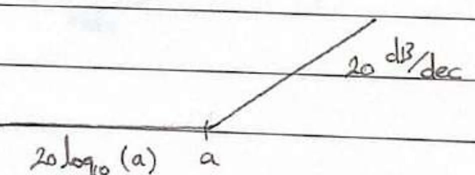
$20\log_{10}\left(\dfrac{k\cdot 1\cdot a\cdot b^2}{c\cdot d}\right)$ → shifting the graph up / down.

Contribution of s (0 @ 0)

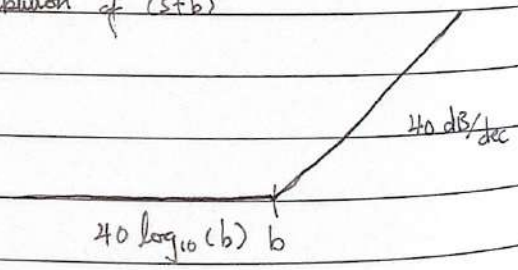


→ Crosses $\omega = 1 \Rightarrow 20\log_{10}(1) = 0$ dB
With $20\log_{10}(\omega_0)$ contribution

Contribution of $(s+a)$



Contribution of $(s+b)^2$



For denominator, just flip the graph upside-down.

For initial contribution (starting point), sum all the contributions from first ω (ω_0) that is not 1. For $H(s)$ above $dB(\omega_0) = 20\log_{10}\left(\dfrac{k\cdot 1\cdot a\cdot b^2}{c\cdot d}\right) + 20\log_{10}(\omega_0)$

Note, decade $= \log_{10}\left(\dfrac{\omega_2}{\omega_1}\right)$

Phasor Plot

Given $H(s) = k \cdot \frac{s(s+a)(s+b)^2}{(s+c)(s+d)}$

$k \Rightarrow 0°$ $\quad s \to 90°/dec$ $\quad 0.1a \Rightarrow 45°/dec$ $\quad 0.1b \Rightarrow 90°/dec$

$s^2 \Rightarrow 180°/dec$ $\quad 10a \Rightarrow -45°/dec$ $\quad 10b \Rightarrow -90°/dec$

$0.1c \Rightarrow -45°/dec$

$10c \to 45°/dec$

Bode - Martini Plot

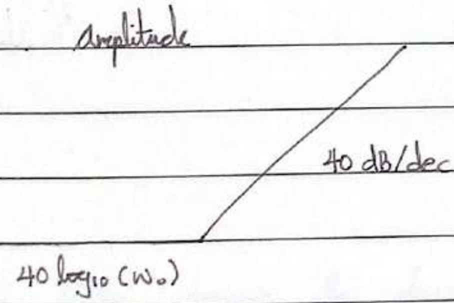
When $H(s)$ has a two cmplx conj, instead of using Bode approx, we make use of Martini approx.

$a+bj$ & $a-bj \to [s+(a+jb)][s+(a-jb)] = s^2 + 2as + a^2+b^2$

$= s^2 + 2\xi\omega_0 s + \omega_0^2$, $a = \xi\omega_0$

ξ (damping freq.), ω_0 (undamped freq.)

Amplitude



Phase

$\log_{10}\left(\frac{\omega_0}{\omega_1}\right) = \xi$ $\quad \log_{10}\left(\frac{\omega_2}{\omega_0}\right) = \xi$

$\omega_1 \Rightarrow 90/\xi$ deg/deg $\quad \omega_2 \Rightarrow 90/\xi$ deg/dec

*watch for denom or numer. Above is numer example.

II Filters:

Resonance, is a freq where L&C cancels out (no reactive element) such that V_{src} only sees resistive component. (Z_T is real only)

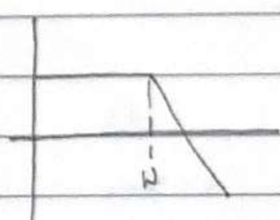
Quality Factor, number ______ that determines the selectivity of the circuit (filter)

Half-Power Freq, ω where the signal is 3dB off of filtered value.

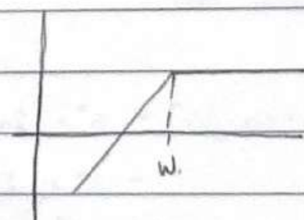
$\omega_0 = \frac{1}{\sqrt{LC}}$ $\qquad Q_s = \frac{\omega_0 L}{R}$ or $\frac{1}{\omega_0 RC}$ $\qquad \underset{(\omega_1, \omega_2)}{\text{HPF}} = \omega_0 \left[\sqrt{1 + \left(\frac{1}{2Q}\right)^2} \pm \frac{1}{2Q} \right]$

$BW = \omega_0/Q$ $\qquad Q_P = \frac{R}{\omega_0 L}$ or $\omega_0 RC$

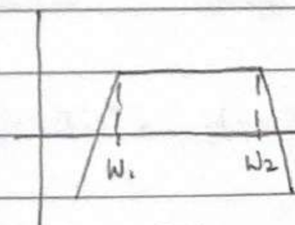
$Q \gg 10 \rightarrow$ highly selective, $(\omega_1, \omega_2) = \omega_0 \pm \frac{BW}{2}$



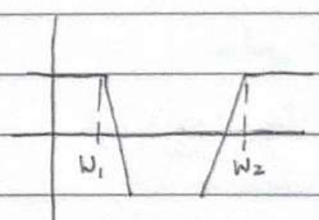
LPF



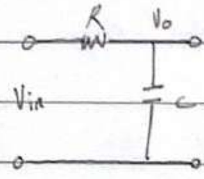
HPF



BPF



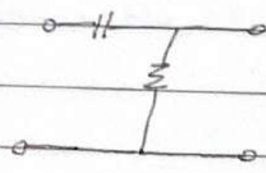
BSF



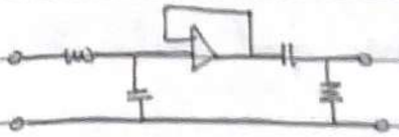
LPF

$\frac{V_o}{V_{in}} = \frac{1}{CRs + 1}$ pole @ $-1/RC$

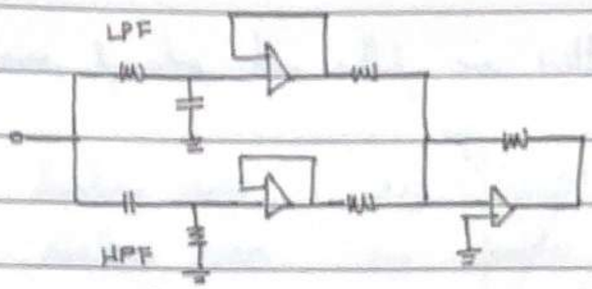
$\omega = \frac{1}{RC}$


HPF

$\omega = \frac{1}{RC}$



LPF cascade HPF → BPF

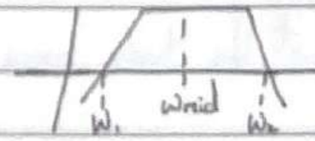


additive op-amp → BSF

Order of filters are determined by the slope of its poles.

Mid-freq. is where filter hits its peak among BPF.

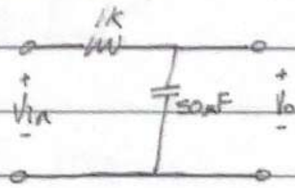
$\omega_{mid} = (\omega_1 \omega_2)^{1/2}$.



Amax & Amin,

Amax is the max level of signal it rejects (ω_p) & Amin is the min level of signal it rejects (ω_s).

example.

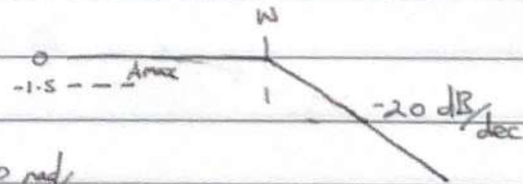


$H(s) = \dfrac{1/sC}{R + 1/sC} = \dfrac{1/RC}{s + 1/RC}$ $\omega = \dfrac{1}{RC}$

$f = 1\,Hz$

$A_{min} = 1.5$

$A_{max} = 15$



$\omega = \dfrac{1}{RC} = 20 \text{ rad/s}$

$H(s) = \dfrac{20}{s+20} \rightarrow 20\log_{10}\left(\left|\dfrac{20}{j\omega + 20}\right|\right) = 0 - 1.5$ ← fsolve.

$\omega_p = 12.85$

$20\log_{10}\left(\left|\dfrac{20}{j\omega + 20}\right|\right) = 0 - 15$

$\omega_s = 110.672$

Sensitivity factor measures the changes a filter experience with a change of variable. Sensitivity $= \dfrac{\omega_p}{\omega_s} < 1$

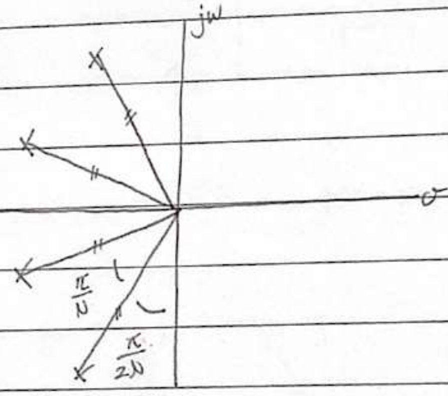
Filter cookbook allows a scaling computation with a given filter.

$R' = k_m R$ $R' = R$ $R' = k_m R$

$L' = k_m L$ & $L' = L/k_f$ → $L' = L\dfrac{k_m}{k_f}$

$C' = C/k_m$ $C' = C/k_f$ $C' = C\dfrac{1}{k_m k_f}$.

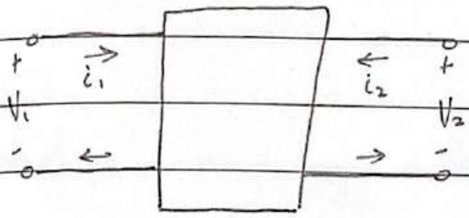
Butterworth Filters are filters with poles of same angle & distance apart.



N - order

x's are point which are equidistant from the origin.

Two - Port Network :



Z_{xy} - Impedance param h_{xy} - hybrid param

Y_{xy} - Admittance param g_{xy} - inverse param.

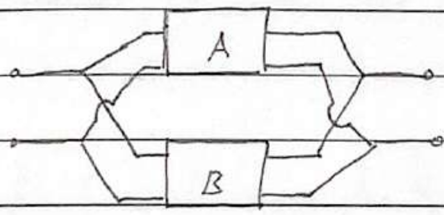
$V_1 = Z_{11} i_1 + Z_{12} i_2$ $i_1 = Y_{11} V_1 + Y_{12} V_2$

$V_2 = Z_{21} i_1 + Z_{22} i_2$ $i_2 = Y_{21} V_1 + Y_{22} V_2$

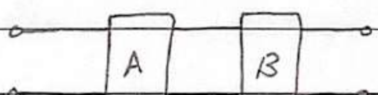
Use param. chart and convert params.

$V_1 = h_{11} i_1 + h_{12} V_2$ $V_1 = A V_2 + B(i_2)$

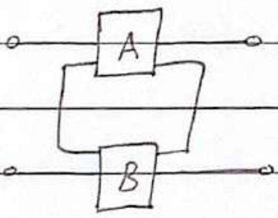
$i_2 = h_{21} i_1 + h_{22} V_2$ $i_1 = C V_2 + -D(i_2)$ {ABCD ⇒ transmission param}



Parallel , $[Y] = Y_A + Y_B$



Cascade , $[T] = T_A \cdot T_B$



Series , $[Z] = Z_A + Z_B$